As you walk through the problem below examine it for the use of the associative property of multiplication which allows us to regroup information to make the process easier and also how the value of the decimal place is expressed as a fraction.

5.27×3.3

$5.27 \times 3.3 = \left(527 \times \frac{1}{100}\right) \times \left(33 \times \frac{1}{10}\right)$ Note how the decimal place is expressed as a fraction.

$5.27 \times 3.3 = 527 \times \left(\frac{1}{100} \times 33\right) \times \frac{1}{10}$ Using the associative property of multiplication you can regroup the factors in the steps that follow.

$5.27 \times 3.3 = 527 \times \left(33 \times \frac{1}{100}\right) \times \frac{1}{10}$ Use the commutative property of multiplication to rearrange.

$5.27 \times 3.3 = (527 \times 33) \times \left(\frac{1}{100} \times \frac{1}{10}\right)$ Here we apply the rules for multiplication of fractions.

$5.27 \times 3.3 = (527 \times 33) \times \left(\frac{1}{1000}\right)$ Now we apply the rules for multiplication of whole numbers.

$5.27 \times 3.3 = 1739\cancel{1} \times \frac{1}{1000}$ In this step, we observe the rules for multiplying a whole number and a fraction.

$5.27 \times 3.3 = \frac{1739\cancel{1}}{1000}$ Lastly we divide as indicated.

$5.27 \times 3.3 = 17.39\cancel{1}$

CONTENT AREA TWO: SCIENTIFIC NOTATION

Scientific notation is a relative newcomer to the mathematical scene. It is thought that physicists working to define the electrical standards such as volt and ohm were the first to develop it as a sort of mathematical shorthand. Scientific notation is a method that allows us to write extremely large or small numbers easily. It is based on the concept of place value expressed in Figure 15.6. The process is rather simple and involves movement of the decimal point right or left and counting. The methodology as laid out is important so that you understand each step in the process so that you can attack multiple-choice or essay questions on the examination effectively. Copy each step and focus on the reason for the action. This will be helpful if you are faced with an essay question that explores scientific notation.

In Figure 15.6, note how the number is transformed by counting the number of moves it takes to get a number into the *ones* place before adding the exponents to account for the zeros you have removed.

FIGURE 15.6

Compacting a Large Number	**Reasoning**
289,500,000,000,000	
2.89500000000000	• Move the decimal to the *left* until you have one digit in front of it.
There are **14** total moves.	• Count your total number of moves left, **14** in this case.
10^{14}	• Use that number as your power of 10 exponent.
2.895×10^{14}	• Write as a product of the decimal number and power of 10. Eliminate zeros as needed.

Now try to apply the same procedure on the number in the practice problem below.

Compact this number into scientific notation: 1,259,000,000,000,000

You should have arrived at 1.259×10^{15}. If you didn't, let's examine how the answer was reached. In every whole number a decimal is at the end whether it is written or not.

We move the decimal to the left until one digit remains in front of it and count as we move and use that as our power for the 10. We moved the decimal until we were left with 1.259 and since we moved a total of 15 places to the left we have 10^{15}.

We can also expand a number out of scientific notation into standard notation (see Figure 15.7).

FIGURE 15.7

To expand a number compacted with scientific notation	**Reasoning**
4.95×10^8	• Reverse the compacting process.
There are **8** total moves.	• Take the power of 10.
4.95 → ↑↑↑↑↑↑↑	• Write the decimal number.
495000000	• Move the decimal to the *right* the same number of places as the power of 10.
495,000,000	• Add commas as needed.

Expand this number into standard notation: 3.925×10^{11}

You should have arrived at 392,500,000,000. Here is how the answer was achieved. The decimal was taken from where it was and moved 11 places to the right because the power of 10 was 11.

The next thing to learn is how to work with decimals and scientific notation. Review how to compact decimals into scientific notation to reflect extremely small fractions of one. Remember that you have to count the number of zeros behind the decimal point and express the exponent negatively (see Figure 15.8).

FIGURE 15.8

Compact Decimal Numbers	**Reasoning**
.0000000000532 →	
5.32	• Move the decimal to the *right* until you have one natural number in front of the decimal. Discard the zeros.
There were **11** total moves.	• Count the total number of moves left you made.
10^{-11}	• Write it as the negative power of ten.
5.32×10^{-11}	• Write it as the product of the decimal number and the negative power of 10.

Compact this decimal number into scientific notation: .000000000000445

You should have arrived at 4.45×10^{-13}. If not, let's examine the process. We moved the decimal to the right until we had one number in front of the decimal, 4.45, and we counted as we moved to determine the power of ten. We moved 13 places so we write the power as 10^{-13}.

Now, let's learn the reverse: How to expand a number in scientific notation into a decimal (see Figure 15.9). Remember to look for negative exponents when deciding the direction in which to add the zeros (either behind or in front of the decimal).

FIGURE 15.9

	2.53×10^{-8}
.00000002.53	• Write the compacted number.
There were **8** total moves.	• Count the negative power of 10.
.0000000253	• Move the decimal that many places to the *left*, adding zeros as necessary.

As you now know, the sign in front of the exponent tells you everything that you need to know about where to add your zeros.

Write the following number in standard notation: 7.85×10^{-12}

You should have arrived at .00000000000785. If not, let's examine how it was worked. The power of 10 tells us the number of places to move the decimal to the ~~right~~ left. I know from the negative power that I will be creating a decimal. I take 7.85 and move the decimal ~~right~~ left 12 places adding zeros as needed.

CONTENT AREA THREE: PROPERTIES OF NUMBERS

Over the centuries people have observed some patterns that have evolved into some basic properties of how rational numbers work. They have been given names—identity, inverse, associative, commutative, and distributive; and they allow us to manipulate numbers, develop equality, and give us methods to prove mathematical principles. The properties seem self-evident mathematically, but in algebra they help us solve equations.

- Identity properties allow us to state positively that a number is what it is.
- Inverse properties of addition show that any number plus its opposite is equal to zero, whereas the inverse property in multiplication allow us to show that any number multiplied by its reciprocal is equal to one.
- Associative properties give us a method of regrouping in addition and multiplication and still have the same result.
- Commutative properties give us the ability to reverse the order of addition and multiplication and still achieve the same answer.
- Distributive properties allow us to spread multiplication over a group of numbers that are added, subtracted, ~~or multiplied.~~

Mathematical models have been standardized to illustrate these properties. A proper knowledge of the properties of numbers allows us not only to understand the functioning of rational numbers, but also to facilitate our understanding of how equations are solved.

Identity

The property of identity allows us to prove that a number is what it claims to be. Remember that 2 + 2 = 4? What that means is that 4 *is* the combination of 2 and 2. These mathematical

TABLE 15.4

Distributive Property across Addition	$A \times (B + C) =$ $(A \times B) + (A \times C)$	$2 \times (4 + 5) =$ $(2 \times 4) + (2 \times 5)$	You can use the Distributive Property to rewrite one factor as the sum of two numbers.
Distributive Property across Subtraction	$A \times (B - C) =$ $(A \times B) - (A \times C)$	$5 \times (6 - 3) =$ $(5 \times 6) - (5 \times 3)$	You can use the Distributive Property to rewrite one factor as the difference of two numbers

In the practice problem below, describe steps to the solution, justify each step, and name the property shown.

Problem	Answer	Justification	Property Illustrated
A. $4 + (-3) + (-6)$			
B. $5 \times 1/5 \cancel{+ 0}$			
C. $5 \times (3 \times 2)$			
D. $7 \times (3 + 9)$			
E. $5 \times (6 - 2)$			

Answers to Practice Problem: (A) ~~5~~ -5 Associative Property of Addition, (B) ~~0~~ 1 Inverse Property of Multiplication, (C) 30 Associative Property of Multiplication, (D) 84 Distributive Property, (E) 20 Distributive Property

CONTENT AREA FOUR: EXPONENTIAL NOTATION

The use of exponents dates from the late 1400s, but it was Rene Descartes who developed the exponent as we use it today. While examining place value and scientific notation, you noticed we used exponents (or powers) of 10. Students often struggle with this type of notation. This confusion can often be traced to a single illustration often used by the teacher to show students the functioning of exponents: 2^2. Even if you work it incorrectly you get the correct answer. Students get the idea that by multiplying the base and the exponent, a correct answer is achieved. If answering an essay question on exponents it would be useful to point this out, as well as the correct illustration:

$\text{BASE}^{\text{exponent}}$

The base is the number that is repeatedly multiplied, the exponent tells us the number of times to use it, so 5^3 means $5 \times 5 \times 5$.

Consider how the base and exponent are related and see if you can devise an essay question that might incorporate your knowledge.

$3 \times 3 \times 3 \times 3$	The exponent tells us how many times to multiply the base.
$(3 \times 3) \times (3 \times 3)$	Group if possible to make your work easier.
81	Multiply

Work through the practice problem for exponential notation, and demonstrate your knowledge of the process.

4^3

You should have arrived at 64. The problem is worked at the top of the next page.

TABLE 15.7

OPERATION	PART NAME	PART NAME	TOTAL	KEY WORDS
Addition $2 + 2 = 4$	Addend 2	Addend 2	Sum 4	Total, together, after, joined
Subtraction $4 - 2 = 2$	Minuend 4	Subtrahend 2	Difference 2	More than, minus, before, profit, less
Multiplication $2 \times 2 = 4$	Factor 2	Factor 2	Product 4	Of, times, double
Division $\begin{array}{r} 1 \\ 2\overline{)2} \end{array}$	Divisor (outside) 2	Dividend (inside) 2	Quotient (above) 1	Each, average

TABLE 15.8

ORDER	MEMORY DEVISE	OPERATION	MEANING	EXAMPLE
1	*Please*	**P**arentheses	Do any operation within the parentheses first.	$(2 + 5)$
2	*Excuse*	**E**xponents	Do any calculation involving and exponent.	2^3
3	*My Dear*	**M**ultiplication and **D**ivision	Do any operation involving multiplication or division.	4×5
4	*Aunt Sally*	**A**ddition and **S**ubtraction	Do any operations involving addition or subtraction.	$5 + 8$

Let us apply the order of operations to this problem (see Figure 15.10).

FIGURE 15.10

Please	**(2 + 5)** + $(2^3) \times 5$
Excuse	*(7)* + **(2^3)** × 5
My	(7) + ***8* × 5**
Dear	No Division
Aunt	**(7) + *40***
Sally	No Subtraction
	47

One additional rule is the *left to right* rule. If all of the operations are the same, then just forget about "Sally" and work through the problem from left to right:

$$\underrightarrow{2 + 3 + 6 + 11 + 67 + 102 + 5} = 196$$

Rounding Off

Rounding off is a method of making numbers more manageable (e.g., when you don't want to operate all that much). With whole numbers, rounding allows for us to look at the big picture in a general way. For decimal numbers, rounding allows us to deal with nonrepeating and repeating decimals. The process is the same for both with a small twist at the end. Please follow and copy each step of the problem in Table 15.9.

TABLE 15.9

ROUND 1255 TO THE NEAREST HUNDRED	REASONING
$1\underline{2}55$	• Underline the place you are rounding off to.
$1\underline{2}55$ $\uparrow$	• Draw an arrow to the next number to the right. Is the number you are pointing at 5 or larger? (yes or no?)
+1 $1\underline{2}55$	• If "yes" add 1 to the underlined number, if "no" leave the underlined number the same.
$1\underline{3}55$	• Is this a decimal number? (yes or no?)
"No"	• If "yes" drop all numbers after the one underlined. If "no" change all numbers after the unlined one to zeros.
1300	

Use the example to complete the practice problem below for rounding.

Round 14579 to the nearest thousand.

You should have arrived at 15,000. Here are the steps that were taken to round off:

14579

$1\underline{4}579$
$\uparrow$

+1
$1\underline{4}579$

15000

Decimal numbers can also be rounded. We do this to eliminate repeating decimals. Repeating decimals are denoted by a bar over the number or pattern that repeats. The repeat goes on forever. For example:

$26.\overline{6}$	means	26.66666666. . . .
$26.\overline{45}$	means	26.45454545. . . .

Rounding decimals is also useful for decimals that are simply too long to deal with. For example, if you saw that the price of a piece of gum was .2513412341341341, you'd probably go crazy. Much easier just to pay 25 cents. Note the only difference is how numbers are handled after the rounding off has occurred (see Table 15.10).

Apply the same information to the practice problem below:

Round 135.75675 to the nearest thousand*th*.

You should have arrived at 135.757. Let's examine how we arrived at that answer.

135.75675

$135.75\underline{6}75$
$\uparrow$

+1
$135.75\underline{6}75$

$135.75\underline{7}$

TABLE 15.10 Round to the nearest tenth

139.9651	• Underline the place you are rounding off to.
139.9651 ↑	• Draw an arrow to the next number to the right. Is the number you are pointing at 5 or larger? (yes or no?)
+1 139.9651	• If "yes" add 1 to the underlined number, if "no" leave the underlined number the same. *(Hint: Don't forget to carry if you have to as in normal addition)*
140	• Is this a decimal number? (yes or no?)
	• If "yes" drop all numbers after the one underlined. If "no" change all numbers after the underlined one to zeros.
140	

Whole Number Operations

Whole number operations include all those wonderful things you learned in elementary school. I'm sure you remember all the endless problems of addition, subtraction, multiplication, and division.

The key to whole number operations is alignment. These operations are dependent upon proper alignment of the place values. One helpful technique is to picture graph paper and imagine one number per square. The "ones" or units are all lined up vertically, then "tens" and so forth. Since multiplication and division are shortcuts some liberties are taken. Note how "carrying" takes place. Again, we know that you know how to do simple math operations like this one; however, many tests with essay questions ask you to write out your thought processes.

25 + 103 + 6	
$\begin{array}{r} 25 \\ 103 \\ \underline{+\ 6} \end{array}$	Align the places values so that like will be in line with like.
5 + 3 + 6 = 14	Add the ones.
	You have made 14. One 10 and 4 ones.
$\begin{array}{r} \mathit{1} \\ 25 \\ 103 \\ \underline{+\ 6} \\ \mathit{4} \end{array}$	Write the 4 in the ones column and carry the 1 to the tens column.
$\begin{array}{r} \mathit{1} \\ 25 \\ 103 \\ \underline{+\ 6} \\ 34 \end{array}$	Add the tens column. You have not created ~~tens~~ hundreds so there is nothing to carry.
$\begin{array}{r} 25 \\ 103 \\ \underline{+\ 6} \\ 134 \end{array}$	Add the hundreds column and write the sum.

Go ahead and apply this process to the practice question on the next page, where you will add a column of numbers and demonstrate your understanding the process at each step.

Add the following: 2351 + 35 + 592 + 7

You should have arrived at 2985. If not go back and review the steps illustrated there as we work through the solution together. First remember that position is everything. All numbers in the ones (units) must be aligned. Placing the addends properly we have:

$$\begin{array}{r} 2351 \\ 35 \\ 592 \\ \underline{+\quad 7} \end{array}$$

We now add the ones, making tens. The result is 15. We write the 5 and carry the 1 to the next column. We continue the process and arrive at the sum of 2985.

Let's now review the reverse process of the subtraction of whole numbers. Observe how in addition we "made tens," but in subtraction we "break tens" and then "give to the needy."

$529 - 53$

$$\begin{array}{r} 529 \\ \underline{-\ 53} \end{array}$$

Align the places values so that like will be in line with like.

$$\begin{array}{r} 529 \\ \underline{-\ 53} \\ 6 \end{array}$$

If you have 9 you can give away 3 from that amount.

$$\begin{array}{r} 4 \\ \cancel{5}29 \\ \underline{-\ 53} \\ 6 \end{array}$$

If you have 2 you cannot give away 5 from that amount. Go to the hundreds column and "give to the needy."

$$\begin{array}{rrr} 4 & & \\ \cancel{5} & {}^{12} & 9 \\ \hline - & 5 & 3 \\ \hline 4 & 7 & 6 \end{array}$$

You now have 12 and can subtract 5 and write the difference.

Now apply the steps from the example above to the practice problem. Remember to be careful with the alignment.

Subtract the following: 4389 − 572

You should have arrived at 3817. If not go back and review the steps illustrated there. Remember that placement is everything. The first number stated, 4389 is placed on top, and the 2 of 572 must be aligned under the 9. If I have 9, I can subtract 2. The result is 7. I can do the same with 8 and 7 with a result of 1. However, with the numbers in the hundreds, I must borrow and give to the needy. If I have 3, I cannot give you 5. That is how you arrive at 3817.

Multiplication. Multiplication of whole numbers can be indicated in a number of different ways:

$5 * 7 \qquad 5(7) \qquad (5)7 \qquad 5 \times 7$

Each gives identical results and has the same meaning: multiply five times seven. With a problem like 256 × 64, we can easily see the advantages of multiplication over repeated addition. Just think for a moment: Without multiplication, you'd need to add 256 and its equation 64 times! That's a lot of work and a lot of scratch paper! Here is the process of multiplication illustrated. Follow the reasoning, so that you can write an essay on multiplying if you are asked to do so.

$$\begin{array}{r} 256 \\ \times\ 64 \\ \hline \end{array}$$

$$\begin{array}{r} {\scriptstyle 22} \\ 256 \\ \times\ 64 \\ \hline 1024 \end{array}$$

Multiply 256 by 4. Make tens as you did for addition. This is your first partial product.

$$\begin{array}{r} 256 \\ \times\ 64 \\ \hline 1024 \\ 15360 \\ \hline \end{array}$$

Multiply 256 × 60. Remember that the 6 is in the tens place. You may compensate by inserting a zero at the end of the partial product.

$$\begin{array}{r} 256 \\ \times\ 64 \\ \hline 1024 \\ +\ 15360 \\ \hline 16384 \end{array}$$

Add the partial products.

Write the product.

Now try multiplying the next example number in the practice problem. Work through it in steps, just in case you have to write it out on the test.

Find the product of 7059 × 68.

You should have arrived at 480,012.

$$\begin{array}{r} 7059 \\ \times\ 68 \\ \hline \end{array}$$

Step 1:

$$\begin{array}{r} 7059 \\ \times\ 68 \\ \hline 56472 \end{array}$$

Step 2:

$$\begin{array}{r} 7059 \\ \cancel{56472} \\ \times\ 68 \\ \hline 56472 \\ +\ 423540 \\ \hline 480{,}012 \end{array}$$

Review the steps of multiplication illustrated there if needed. One thing to pay special attention to in the problem is the multiplication with zero and the addition of the number carried to the zero; if a mistake has been made that will be the place to look first.

Division. Division is often difficult for children to grasp because of the numerous steps involved. It can be indicated in a number of different ways:

$12 \div 3$ $\qquad \frac{12}{3}$ $\qquad 3\overline{)12}$

It involves the skills of estimation, multiplication, and subtraction and certain processes unique to division.

For the rest, you will have to perform division as part of larger questions that may involve several steps (e.g., word problems, tables, graphs, etc.), which should not be a problem because you will (in most cases) have a calculator.

The only challenge may be if they ask you to *estimate* the answer and select from a number of options:

> Estimate the answer to the following: 15,667 ÷ 1,333
>
> A. 8
> B. 9
> C. 10
> D. 7

See page 447

To figure the answer out quickly, look at the answers and hope that you see ten. 10 × 1,333 = 13,000, and 13,000 will divide into 15,666 with a remainder that is less than the divisor. Therefore, the answer is C. Each of the other options will yield remainders that are greater than the divisor, so they are out.

Decimal Operations. Decimal operations mirror whole number operations with some minor twists involving placement of the decimal. If it has been a while since you've worked with decimals, then you may long for simple examples. Let's begin with simple addition.

Addition and Subtraction of Decimals. Once again alignment is the key to success. Watch how the place values are lined up. Be sure to align the decimals vertically and add or subtract as if they were whole numbers.

Add 2.3 + .0067 + 364.023

2.3 .0067 + 364.023	Align the decimals so that all place values are to be used like to like. Add "ghost" zeros as needed.
2.3*000* *(ghost terms)* .0067 + 364.023*0*	Add each column as in whole number addition carrying across the decimal.
2.3*000* .0067 + 364.023*0* 366.3297	Be sure that the decimal is aligned in the answer (sum).

> Add 47.006 + .56 + 8.1

You should have arrived at 55.666. If you did not achieve that answer review the steps presented in the example and double check your alignment of decimals and your use of ghost terms in the solution that follows:

```
  47.006
    .56
+  8.1
--------
  47.006
    .560
+  8.100
--------
  55.666
```

Let's learn the reverse process now: subtraction. You will see that the process for alignment is the same as for addition of decimals and that subtraction occurs as it does with whole number subtraction.

	Subtract 23.08 − .0352
23.08 − .0352	Align the decimals so that all place values are to be used like to like. Add ghost zeros as needed.
23.08*00* − .0352	Subtract each column borrowing as necessary as in whole number subtraction.
23.08*00* − .0352 23.0448	Be sure that the decimal is aligned in the answer (difference).

Subtract 25.08 − .8023

You should have arrived at 24.2777. If you did not achieve that answer, review the steps presented and double check your alignment of decimals and use of "ghost" terms. Also reexamine your borrowing across zeros. The solution is presented below:

$$\begin{array}{r} 25.08\mathit{00} \\ -\ 00.8023 \\ \hline 24.2777 \end{array}$$

Multiplication of Decimals. Unlike in addition and subtraction, alignment does not matter when multiplying decimals. You will find that you multiply as with whole numbers. Once you have an answer (product), count the number of places behind the decimal in the factors and move the decimal from right to left in the product the same number of places.

	Multiply 28.52 × .05
28.52 × .05	Write the problem. There is no reason to align the decimals.
28.52 × .05 14260	Write the product.
28.52 × .05 14260	Add up the places behind the decimal in the factors.
28.52 × .05 1.4260	Move the decimal from right to the left the same number of places.

Multiply 26.03 × .525

You should have arrived at 13.66575. If you did not achieve that answer review the steps presented and double-check your movement of the decimal in the answer. Remember in

multiplication there is no need to align decimals. Double-check your multiplication and carrying across zeros. Here is the solution worked out for you:

$$\begin{array}{r} 26.03 \\ \times\ .525 \\ \hline 13015 \\ 52060 \\ +\ 1301500 \\ \hline 13.66575 \end{array}$$

Division of Decimals. Having reviewed multiplication of decimals you will now be reminded of the steps of division. You discovered that you placed the decimal as the last step in the process. Division is the inverse of multiplication. Logically then you would deal with the decimal first. The important thing to remember is that you must have a whole number in the divisor. If it is a decimal number move the decimal to the right until it is next to the division brace. Then move the decimal the same number of places under the division brace, adding zeros as necessary.

~~Divide~~ Try 3.15 ÷ .03

.03)3.15	Write the problem using the division brace.
3)3.15.	Move the decimal in the divisor until it is next to the division brace.
3)315	Move the decimal under the division brace the same number of spaces adding zeros if necessary.
105 over 3)315	Divide as you would with whole numbers, adding zeros as necessary until the problem has a zero remainder or until the point you are being asked to round off to or the decimal repeats.

Divide 32.882 ÷ .02

You should have arrived at 1644.1. If you did not achieve that answer, review the steps and double-check your movement of the decimal in the divisor. ~~Other~~ Another problem area may be placement of the numbers in the initial setup of the problem. Remember the placement of the numbers is crucial. The number you are dividing by (the number following the ÷) is placed outside the division brace.

CONTENT AREA SIX: FRACTIONS

The key to all fractional operations is the number 1. One is defined as any number over itself such as:

$$\frac{2}{2} \qquad \frac{550}{550}$$

One is what allows us to reduce fractions, write equivalent fractions, change to common denominators, "cross-cancel" in multiplication, and to divide. The top number of a fraction is called the *numerator* and names how many parts of the fraction that you have. The bottom number of a fraction is called the *denominator,* and it denotes how many total pieces of the fraction are available. The various types of fractions are discussed here.

Proper Fractions

Fractions where the numerator is *smaller* than the denominator are referred to as "proper." Some examples are

$$\frac{7}{8} \qquad \frac{2}{3} \qquad \frac{3}{4}$$

Improper Fractions

Fractions where the numerator is *larger* than the denominator are referred to as "improper." Some examples are

$$\frac{12}{5} \qquad \frac{3}{2} \qquad \frac{5}{3}$$

To change an improper fraction to a mixed number, divide the numerator by the denominator. The answer (quotient) becomes the whole number, the remainder becomes the numerator of the fraction, and the number you have divided by (divisor) is the denominator of the fraction.

Example: Write $\frac{15}{4}$ as a mixed number.

$$\frac{15}{4} = 4\overline{)15}$$

The answer would be 3 with a remainder 3. The number you divided by was 4, so you write the mixed number as:

$$3\tfrac{3}{4}$$

Therefore, the mixed number would be $3\frac{3}{4}$.

Mixed Numbers

As you just witnessed, mixed numbers are a combination of whole numbers and fractions. Some examples are $1\frac{3}{4}$, $4\frac{3}{5}$, and $7\frac{1}{5}$. In multiplication and division of fractions, mixed numbers must be converted to fractions before use. To change a mixed number to a fraction, multiply the denominator by the whole number and add to the numerator (bottom times side, add to top). Then write over the denominator.

Write $3\frac{3}{4}$ as an improper fraction.

$4 \times 3 = 12,\quad 12 + 3 = 15.$ The improper fraction would be $\cancel{\frac{15}{4}}$. $\frac{15}{4}$

Equivalent Fractions

Equivalent fractions are fractions that name the same amount. For example:

$$\frac{2}{3} \qquad \frac{4}{6}$$

They may appear to be different, but when these fractions are compared, they are identical. Another way to think about it is this: It makes no difference if you ate 2 out of 3 slices of pizza or 4 out of 6 slices, because you still have the same amount of pizza.

Here is a trick to test to see if factions are equal or not:

$$\frac{2}{3} \quad \frac{4}{6}$$

Multiply denominator to numerator and denominator to numerator again:

$3 \times 4 = 12$; $\quad 6 \times 2 = 12$. $\quad$ Thus, they are both equal.

To discover if two fractions are equivalent, you will cross-multiply and compare the results. If equal, the fractions are equivalent. If not, one will either be greater than (>) or less than (<) the other. Let's see which fraction is larger here:

$$14 \quad \frac{2}{3} \quad \frac{4}{7} \quad 12$$

Which fraction is larger? $\frac{2}{3}$. How do you know? Because when you multiply 7×2, you get 14; when you multiply 3×4, you get 12. Since 14 arrives on the other side of two-thirds, it is the larger fraction. (*Note:* Be sure to go in the right direction when cross multiplying, or you'll mess up your answer. The order is always denominator to numerator for both fractions!)

Reducing Fractions

To reduce fractions you need to remove all the fractions of one. One method is to divide the numerator and denominator by the greatest common factor. Another method when the common factor isn't obvious is to factor to primes and remove all fractions of one.

	Reduce $\frac{36}{48}$
$\frac{36}{48} = \frac{2\times2\times3\times3}{2\times2\times2\times2\times3}$	Factor the numerator and denominator to primes.
$\frac{36}{48} = \frac{\cancel{2}\times\cancel{2}\times3\times\cancel{3}}{\cancel{2}\times\cancel{2}\times2\times2\times\cancel{3}}$	Cancel the common factors, until you can cancel no more.
$\frac{3}{2\times2}$	Multiply any remaining factors.
$\frac{3}{4}$	Your answer is the reduced fraction.

Reduce $\frac{28}{36}$

The correct answer would be $\frac{7}{9}$. When factoring to primes you would have $\frac{2\times2\times7}{2\times2\times3\times3}$. You will note that there are two examples of fractions of one: $\frac{2}{2}$ and $\frac{2}{2}$. Therefore we have $\frac{7}{3\times3}$. Multiplying we are left with $\frac{7}{9}$.

Multiplication of Fractions

Multiplying fractions is simple: *top* $\times$ *top and bottom* $\times$ *bottom*. But let's qualify "simple." You can only multiple *fractions!* If the number is mixed, then you have to change it to an improper faction, then top $\times$ top and bottom $\times$ bottom. For example:

$$\frac{3}{7}\times\frac{2}{5}=\frac{6}{35}$$

Once this step is completed the pattern is simple, just like we said it was: *top* × *top and bottom* × *bottom*. This can get complicated if the numerators and denominators are really large, as in:

$$\frac{120}{234}\times\frac{140}{456}=\frac{16800}{106704}$$

As you can see, multiplying 120 × 140 and 234 × 456 will give you really large numbers and take a large amount of time. That's why we have "cross-canceling." It allows you to reduce the fractions before you multiply thus saving time and reducing the potential for mistakes (see Cross-Canceling Common Factors). (Next Page)

	Multiply $\frac{2}{3}\times\frac{1}{5}$
$\frac{2\times 1}{3\times 5}$	Writing horizontally often reduces mistakes.
$\frac{2\times 1=2}{5\times 3=15}$	top × top (numerator × numerator) bottom × bottom (denominator × denominator)
$\frac{2}{15}$	Write the answer in simplified (reduced) form.

Now, transfer this knowledge to the problem below.

Multiply $\frac{1}{3}\times\frac{2}{7}$

The answer is $\frac{2}{21}$. If you did not get that answer, go back and review the master pattern for multiplication of fractions. Remember the pattern: top times top; bottom times bottom. Here is the solution worked for you:

$$\frac{1\times 2}{3\times 7}=\frac{2}{21}$$

Fractions with Mixed Numbers and Whole Numbers. To offer you a challenge, the next example shows you how to multiply fractions with mixed numbers. Remember: You can only multiply fractions, so all mixed numbers and whole numbers must be converted to fractions before use in multiplication.

	Multiply $1\frac{1}{4}\times\frac{1}{3}\times 7$
$1\frac{1}{4}$ is a mixed number	Change a mixed number into a fraction by multiplying the denominator by the whole number and adding to the numerator. Place it over the denominator.
$(4\times 1)+1=5$	
$\frac{5}{4}$	
$\frac{7}{1}$	Whole numbers must be converted to fractions before multiplying.
$\frac{5}{4}\times\frac{1}{3}\times\frac{7}{1}$	
$5\times 1\times 7=35$	Multiply the numerators.
$4\times 3\times 1=12$	Multiply the denominators.

$\frac{35}{12}$	Write as a fraction (improper in this case).
$2\frac{11}{12}$	Change improper fraction to mixed number.

As you do the practice problem, carefully check to see if you have mixed numbers or whole numbers and be sure you change them to fractions first.

Multiply $2\frac{1}{3} \times \frac{1}{4} \times 7$

If you answer was $4\frac{1}{12}$, you did the work correctly. If your answer was $\frac{49}{12}$, you didn't change it from an improper fraction to a mixed number. This is the problem worked step by step:

$$2\frac{1}{3}\times\frac{1}{4}\times 7 \qquad \frac{7}{3}\times\frac{1}{4}\times\frac{7}{1} \qquad \frac{7}{3}\times\frac{1}{4}\times\frac{7}{1}=\frac{49}{12} \qquad 12\overline{)49} \qquad 4\frac{1}{12}$$

12 goes into 49 four times with a remainder of 1. Write the remainder over the divisor, which gives you $\frac{1}{12}$. Now using 4 as your whole, write the mixed number as $4\frac{1}{12}$.

Cross-Canceling Common Factors. You will recall that when we reduced fractions, we factored the numerator and denominator and "canceled" any pairs of numbers found on the top and the bottom. When we multiply fractions, we can apply the same principle to make them easier to work with. Note once again how the number 1 is woven through fraction operations. (Reminder: All mixed numbers and whole numbers must be expressed as fractions.)

	Multiply $\frac{14}{15}\times\frac{5}{7}$
$\frac{2\times 7}{3\times 5}\times\frac{5}{7}$	Factor the numerators and denominators to primes.
$\frac{2\times \cancel{7}}{3\times \cancel{5}}\times\frac{\cancel{5}}{\cancel{7}}$	Identify the fractions of one.
$\frac{2\times 1}{3\times 1}\times\frac{1}{1}$	Replace the fractions of one with 1.
$\frac{2}{3}$	Follow the pattern of multiplication.

As you can see, cross-canceling makes finding the reduced answer so much easier.

Addition and Subtraction of Fractions

You probably remember that when you add fractions, you have to make the denominators the same. For example, you can add (or subtract)

$$\frac{3}{5}+\frac{4}{5}$$

to get $\frac{7}{5}$, but you cannot add (or subtract)

$$\frac{3}{5}+\frac{4}{7}$$

yet, because the denominators are not the same. The process to set up fractions for addition and subtraction are the same. The rule in the most basic language is this: When the bottoms are the same you can add or subtract the tops. If the bottoms are not the same, you must

find a new bottom. Mathematically it can be stated: When the denominators are the same, you may add or subtract the numerators. If the denominators are not the same, *then you must find a common denominator.* If you know the concept of fractions of one and can count, you have all the skills needed to add or subtract fractions. As you follow the process presented next, note how we use the identity property of multiplication to guarantee that the fraction we end with has the exact same value as the fraction we start with.

Add $\frac{3}{4}+\frac{2}{3}$

$$\begin{array}{r} \frac{3}{4} \\ +\ \frac{2}{3} \\ \hline \end{array}$$

Rewrite the problem in an easier form to work with.

$$\begin{array}{rr} \frac{3}{4} & 12 \\ +\ \frac{2}{3} & 12 \\ \hline \end{array}$$

The denominators are not the same, so a new common denominator must be located. *Hint:* When in doubt count by the smaller until the larger will divide it.

$$\begin{array}{rr} \frac{3}{4}\times\frac{3}{3}= & 12 \\ +\ \frac{2}{3}\times\frac{4}{4}= & 12 \\ \hline \end{array}$$

Ask yourself, $4 \times ? = 12$ and create a fraction of one. Do the same for the second fraction.

$$\begin{array}{r} \frac{3}{4}\times\frac{3}{3}=\frac{9}{12} \\ +\ \frac{2}{3}\times\frac{4}{4}=\frac{8}{12} \\ \hline \end{array}$$

Multiply the numerators.

Multiply the denominators.

$$\begin{array}{r} \frac{3}{4}\times\frac{3}{3}=\frac{9}{12} \\ +\ \frac{2}{3}\times\frac{4}{4}=\frac{8}{12} \\ \hline \frac{17}{12} \end{array}$$

The denominators are now the same so the numerators may be added.

$$\frac{17}{12}$$

Is the answer a proper or improper fraction? If proper, reduce if possible.

improper $17 \div 12$ — If improper, make into a mixed number and reduce if possible.

$$1\frac{5}{12}$$

Add $\frac{5}{6}+\frac{3}{4}$

17/12

The result should be $1\frac{7}{12}$. If your answer differed, let's examine the solution:

$$\begin{array}{rrrr} \frac{5}{6} & \frac{5}{4}\times\frac{4}{4}=\frac{20}{24} & & \\ +\ \frac{3}{4} & +\ \frac{3}{4}\times\frac{6}{6}=\frac{18}{24} & \frac{38}{24} & 1\frac{7}{12} \\ \hline \end{array}$$

Subtraction of Fractions. Subtraction of fractions sets up identically to addition. You simply subtract the numerators. There is a twist when it comes to borrowing. Just remember this

The FOIL Method of Multiplication of Polynomials

FOIL is the second method of multiplication of polynomials. It's an acronym for a pattern:

First	First Term	×	First Term
Outer	Outer Term	×	Outer Term
Inner	Inner Term	×	Inner Term
Last	Last Term	×	Last Term

Following the multiplication, you add the like terms of the partial products.

$(a + 3)(a - 5)$	First Term × First Term	a^2		
	Outer Term × Outer Term		$-5a$	
	Inner Term × Inner Term		$3a$	
	Last Term × Last Term			-15
	Combine Like Terms	a^2	$-2a$	-15

Division of Polynomials

The easiest type of problem in division with polynomials is when you divide by a monomial (single term). The reason being is that you can set it up as a fraction, and visually it is easier to work. The trick is to remember that each term on the top (numerator) is divided by the one on the bottom (denominator). For example:

$$(8a^2 - 4a) \div 2a = \frac{8a^2 - 4a}{2a} = \frac{8a^2}{2a} - \frac{4a}{2a} = 4a - 2$$

As you can see, $(8a^2 - 4a) \div 2a$ becomes a simple fraction that we can work with:

$$\frac{8a^2 - 4a}{2a}$$

Now, remember all of those properties that you memorized earlier in domain one? This is the distributive property. This property lets us simplify the equation even further:

$$\frac{8a^2}{2a} - \frac{4a}{2a}$$

Look at all those numbers that are divisible by $2a$! See? Now you can cancel and reduce until you are left with

$$4a - 2$$

With tricks like these, you'll have no problem on the test.

$$(9b^3 + 18b^2) \div 3b^2$$

$\frac{9b^3 + 18b^2}{3b^2}$	A division problem can be rewritten in fractional form.
$\frac{9b^3}{3b^2} + \frac{18b^2}{3b^2}$	Each term is to divided by the denominator so the problem can be separated into its individual components.
$3b + 6$	Divide each term by the denominator.

$(12x^5 - 9x^4) \div 3x^3$

The answer to the practice problem is $4x^2 - 3x$. Did you divide each coefficient by 3? Did you subtract the exponents of the x terms?

Here are the steps to solution:

$\frac{12x^5}{3x^3} - \frac{9x^4}{3x^3}$	Rewrite the division problem in fractional form.
$4x^2 - 3x$	Divide each term by the denominator.

Division of a Polynomial by a Polynomial. Division of a polynomial by a polynomial is a little more complicated than dividing by a monomial, but the format will seem very familiar and it will help make the process easier. As with so many operations, with polynomials you will have to be careful with your signs. Remember that when you subtract, signs must be changed because the minus sign has been redefined as "plus the opposite of." One additional caution: You may have to add in "ghost" terms to hold places in the problem as in the example.

To work these problems you will use your old friend from elementary school, the division brace.

$(25a^2 + 15a + 25) \div a + 5$

$$a + 5\overline{)25a^2 + 15a + 25}$$

First write the equation in standard division format. Remember the number you are dividing by (the one after the sign of division) goes on the outside of the division brace. For the first step ask yourself, "What can you multiply by to get rid of $25a^2$?" $5a$. Write that directly above the $25a^2$. Then multiply $5a(a + 5)$ using the Distributive Property.

$$\begin{array}{r} 5a \quad - 10 \\ a + 5\overline{)25a^2 + 15a + 25} \\ \cancel{25a^2 + 25a} \qquad \\ \underline{+-25a^2 - 25a} \qquad \\ -10a + 25 \end{array}$$

Write your result directly under $25a^2 + 15a$ and subtract. Don't forget that subtraction will change the signs, create $+ -25a^2 -25a$. Do the math and bring down like in whole number division.

$$\begin{array}{r} 5a \quad - 10 \\ a + 5\overline{)25a^2 + 15a + 25} \\ \cancel{25a^2 + 25a} \qquad \\ \underline{+-25a^2 - 25a} \qquad \\ -10a + 25 \\ -10a-50 \rightarrow \cancel{-10a + 20}\ 50 \\ \underline{+10a + 50} \\ \cancel{25} \\ 75 \end{array}$$

Now we repeat the process. Ask yourself "What can you multiply by to get rid of $-10a$?" The answer would be -10. Why -10? Because you have to take into consideration the sign change when you subtract and it now creates $+ 10a - 50$. Write -10 above $15a$ in the division brace and using the Distributive Property multiply $-10(a + 5)$. Write it under $-10a + 25$ and don't forget that subtraction will change the signs because subtraction in algebra means "plus the opposite of." Write the result as you would with whole number division. You will note we have a remainder.

Write a remainder in fractional form. $\dfrac{\overset{75}{\cancel{-25}}}{a+5}$

The answer is stated as $\cancel{5a - 10 \frac{-25}{a+5}}$. $5a - 10 + \dfrac{75}{a+5}$

In the following example, you will see how ghost terms are inserted into polynomial division as explained above.

Divide: $(a^3 - a) \div (a + 1)$

$a + 1\overline{)a^3 - a}$ — Rewrite the problem using the division brace.

$a + 1\overline{)a^3 + 0a^2 - a}$ — Notice that there is no a^2 term. A ghost term must take its place, $0a^2$.

$$\begin{array}{r} a^2 \\ a + 1\overline{)a^3 + 0a^2 - a} \end{array}$$

What would I multiply a by to eliminate the a^3? a^2. Write it above the a^3.

$$\begin{array}{rrrr} & a^2 & & \\ a + 1) & a^3 & + 0a^2 & - a \\ & \cancel{a^3} & \cancel{+ a^2} & \\ & -a^3 & - a^2 & \\ \hline & & -a^2 & \end{array}$$

Multiply. Now, remembering that the minus sign means "plus the opposite of" change the necessary signs and add.

$$\begin{array}{rrrr} & a^2 & & \\ a + 1) & a^3 & + 0a^2 & - a \\ & \cancel{a^3} & \cancel{+ a^2} & \\ & -a^3 & - a^2 & \\ \hline & & -a^2 & - a \end{array}$$

Bring down as you would in regular division.

$$\begin{array}{rrrr} & a^2 & & - a \\ a + 1) & a^3 & + 0a^2 & - a \\ & \cancel{a^3} & \cancel{+ a^2} & \\ & -a^3 & - a^2 & \\ \hline & & -a^2 & - a \\ & & a^2 & + a \\ \hline & & & 0 \end{array}$$

What would you multiply a by to eliminate a^2? $-a$. Why $-a$? Because you have to take into consideration the sign change when you subtract, and it now creates $a^2 + a$.

$a^2 - a$ — Check the remainder and, if there is one, write it in fractional form using the appropriate sign.

$(n^2 + 2n + 4) \div (n + 1)$

You should have arrived at $n + 1 + \frac{3}{n+1}$. Did you place the term after the $\div$ outside the division brace? Did you insert ghost terms as needed? Did you consider how to eliminate only the first term? Did you change signs of all terms as you subtracted (step 4 above). Did you repeat these steps as needed? Here is the problem worked step by step for you:

$$\begin{array}{rrrr} & n & + 1 & \\ n + 1) & n^2 & + 2n & + 4 \\ & \cancel{n^2} & \cancel{+ n} & \\ & -n^2 & - n & \\ \hline & & n & + 4 \\ & & \cancel{n} & \cancel{+ 1} \\ & & -n & - 1 \\ \hline & & & 3 \end{array}$$

$$n + 1 + \frac{3}{n+1}$$

Factoring Polynomials That Are the Difference of Two Squares

The difference of two squares is like having twins. One twin is "good" (positive) and the other is "bad" (negative). You can always recognize the difference of two squares in factoring because the terms are perfect squares separated by a minus sign. The examples below will help clarify.

$n^2 - 36$
$a^2 - b^2$
$25y^2 - 81$

In each sample, the first and the last term are perfect squares. Each of the perfect squares is separated by a minus sign. Once you can identify them as the difference of two squares, the pattern to solution is always the same. Notice the pattern that develops when we factor the examples above:

The "good twin" is	and	the "evil twin" is
$(n + 6)$		$(n - 6)$.
$(a + b)$		$(a - b)$.
$(5y + 9)$		$(5y - 9)$.

As you can see the terms are identical except that one is positive and the other negative.

$36b^2 - 9n^2$

You should have arrived at $(6b + 3n)(6b - 3n)$. If not, did you examine each term to see if each were a perfect square?

$36b^2$ is a perfect square $36 = (6)(6)$ and $x^2 = (x)(x)$
$9n^2$ is a perfect square $9n^2 = (3)(3)$ and $n^2 = (n)(n)$
They are separated by a minus sign, so they are the difference of two squares and there is a "good" twin and an "evil" twin: $(6b + 3n)(6b - 3n)$

Factoring a Polynomial with Three Terms

Factoring a polynomial with three terms is "unfoiling." You are given the answer and it will be your job to discover the first, outer, inner, and last terms and to determine what sign of operation (addition or subtraction) to be used. There are some basic patterns that will make your life easier. Consider these polynomials of three terms to see what patterns develop:

$a^2 + 8a + 15$

Because all of the terms are positive, I know that the pattern is $(a + 5)(a + 3)$. How were the numbers 3 and 5 arrived at? Ask yourself, "What two numbers when you multiply them give the answer of 15, but when added result in 8?" The only two numbers that meet that condition are 5 and 3. You will note that all terms are added in the original equation; therefore, all terms are added when the factoring is done.

Now consider this equation:

$a^2 - 8a + 15$

The second term is negative and the third term is positive. If you recall the rules for multiplication of signed numbers, you will recall that there are only two ways to multiply and have a positive result (+)(+) or (–)(–). If both terms were positive then the sign between the first and second terms would be positive. That isn't the case here. Further, if you recall the rules for

addition of signed numbers, you will recall that only a negative plus a negative results in a negative. So, when you factor both numbers, the sign must be negative. The pattern in this case would be:

$(a - 5)(a - 3)$

Finally, consider this equation:

$a^2 + 2a - 15$

You will note that in this equation the second term is positive and the last is negative. This calls for an additional strategy. Focus first on the third term: 15. Here, you must ask yourself, "What two numbers if multiplied give 15 for an answer, but if added or subtracted result in 2?" Answering that question will give you the correct sign to use. There are only two possible pairs of factors that will result in 15: 1 and 15, and 3 and 5. Only one of those pairs is two apart, 3 and 5. Now look at the second term. If it is positive, then the larger number of your chosen pair is positive; if negative, the larger number is negative. In this problem your pattern would be $(a + 5)(a - 3)$.

$a^2 - 2a - 15$

As in the previous example, you must ask yourself, "What two numbers multiplied give you –15, but if added or subtracted result in –2?" Answering that question will give you the correct sign to use. Of the two possible pairs of factors that result in 15 only two are apart, 3 and 5. Is the second term negative? If so then the larger of the two factors must be negative. The pattern would be $(a - 5)(a + 3)$.

$b^2 - b - 20$	
$b^2 - b - 20$	The first term must be b since $(b)(b) = b^2$.
$(b \quad)(b \quad)$	
$b^2 - b - 20$	The only pair of factors that make 20 and are 1 apart are 4 and 5.
$(b \quad 4)(b \quad 5)$	
$b^2 - b - 20$	Since the middle term is negative, then the largest of the factors of
$(b + 4)(b - 5)$	20 must be negative.

$x^2 + 4x - 21$

Here is how the solution is reached:

$x^2 + 4x - 21$	
$(x \quad)(x \quad)$	Did you see how x^2 contributes an x to each factor?
$(x \quad 7)(x \quad 3)$	Did you ask yourself what two numbers multiply to make 21, but have a difference of ~~3~~? 4
$(x + 7)(x - 3)$	Did you note in the original problem the $4x$ was positive and the 21 was negative? The only way to get a negative when multiplying is to have opposite signs (positive and negative). The middle term of the original was positive so the larger of the two numbers factored (7 and 3) must be positive.

The Quadratic Equation

Some polynomials are difficult to factor, because we cannot take them apart. To solve difficult polynomials, we must use the quadratic formula (aka quadratic equation) to do so. Let's see how we can actually do this and learn about parabolas while we're at it.

First, here is the standard form for the quadratic formula. You should memorize it.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You may run into the term *discriminant* on the test. Just in case, here is what it is. See that "stuff" inside the $\sqrt{\ }$? The "stuff" beneath the radical is the *discriminant*. You may also be asked about what the significance is of the discriminant. Well, the discriminant (whatever number you get from taking the square root of ~~$b^2 - 4a$)~~ will tell you the type and number of answers that the quadratic formula will produce. There are three types of discriminant possibilities for you to memorize, just in case you are asked questions about them on the test: → $b^2 - 4ac$

- If the discriminant turns out to be positive, then you will have two solutions when you solve.
- If the discriminant is zero, then you will have one solution.
- If the discriminant is negative, then there are no real number answers to the equation.

Now, if you are asked to apply the quadratic formula to a polynomial, or if you find that you cannot factor a polynomial very easily, then use the steps below to execute the quadratic formula. It may help you solve polynomials more easily. Consider the polynomial below:

$$2x^2 = -x + 6$$

First, ask yourself: "Is this equation in the correct form of $ax^2 + bx + c = 0$?" If your answer is "no," what must you do to put it in the proper form? In the case of $2x^2 = -x + 6$, you must move the $-x$ and the 6 to the other side of the equals sign. This is done in the normal manner as previously explained in this domain. You would add x to both sides of the equation and subtract 6. These two moves result in:

$$2x^2 + x + (-6) = 0$$

Understand that $2x^2 + x + (-6) = 0$ is the same as $ax^2 + bx + c = 0$. Here is how each equation lines up:

$$\begin{array}{ccccccc} 2x^2 & + & x & + & (-6) & = & 0 \\ ax^2 & + & bx & + & c & = & 0 \end{array}$$

Here, $a = 2$; $b = 1$ (if there's no number in front of the letter, its always one); and $c = -6$. These letters can now be plugged into the quadratic equation.

Second, it's time to "plug and play." Take the numbers that represent a, b, and c, the polynomial, and plug them into the quadratic formula.

Third, multiply everything inside the radical:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ becomes } x = \frac{-1 \pm \sqrt{1^2 - 4(2) \times (-6)}}{2(2)}$$

Fourth, add inside the radical: $x = \dfrac{-1 \pm \sqrt{1 - (-48)}}{4}$.

Fifth, arrive at the answer: $x = \dfrac{-1 \pm \sqrt{49}}{4}$.

Recall the information on the discriminant. Because the discriminant (the number under the radical) is positive (49), you will have two answers.

Sixth, take the square root of 49: $x = \dfrac{-1 \pm 7}{4}$.

Seventh, show both aspects of the ± sign. To do so, separate the answer into two expressions:

$x = \frac{-1+7}{4}$ *and* $x = \frac{-1-7}{4}$

Finally, we solve for each possible answer:

$\frac{-1+7}{4}$ *or* $\frac{6}{4} = \frac{3}{2}$ *and* $\frac{-1-7}{4}$ *or* $\frac{-8}{4} = -2$

Thus, $x = \frac{3}{2}$ *and* –2.

Find the values for x in the equation below.

$-24x + 45 = -3x^2$

Your answers should be $x = 5$ and $x = 3$. If your results differ, go back and check your solution step by step and refer to the sample problem above.

Before we leave the joys of quadratic equations, we need to end with a brief discussion of parabolas. Parabolas can either be up like a capital U, or down like an upside-down U. The sign of x^2 changes the direction of the parabola. If the sign is negative then it is upside down; if the sign is positive, then it is right side up.

Figure 16.1 shows a graph of the function $y = x^2 - 3$. Since x^2 is positive, the graph is right-side up. Notice how it is like a diver entering a pool, and at the bottom he surges to the top. The bottom of the graph shows the lowest point the diver reaches. This is the minimum or *lower bound* of the graph. A parabola that opens upwards shows a minimum value.

Figure 16.2 shows a graph of the function $y = -x^2 - 3$. Since $-x^2$ is negative, the graph is upside down.

This graph also shows a maximum value. It is like a quarterback throwing a pass. The highest point the ball reaches is the *upper bound,* or maximum value. A parabola that opens down shows a maximum value.

Algebraic Fractions

Surprisingly enough, fractions in algebra are actually easier that with everyday fractions. All the skills you have with them apply, but you no longer have to worry about mixed numbers,

FIGURE 16.1

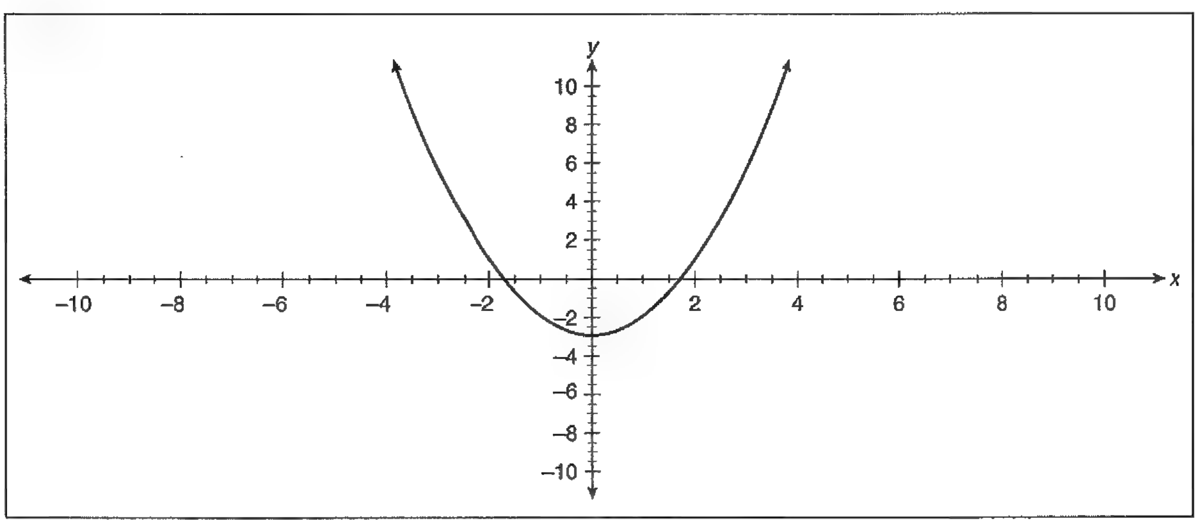
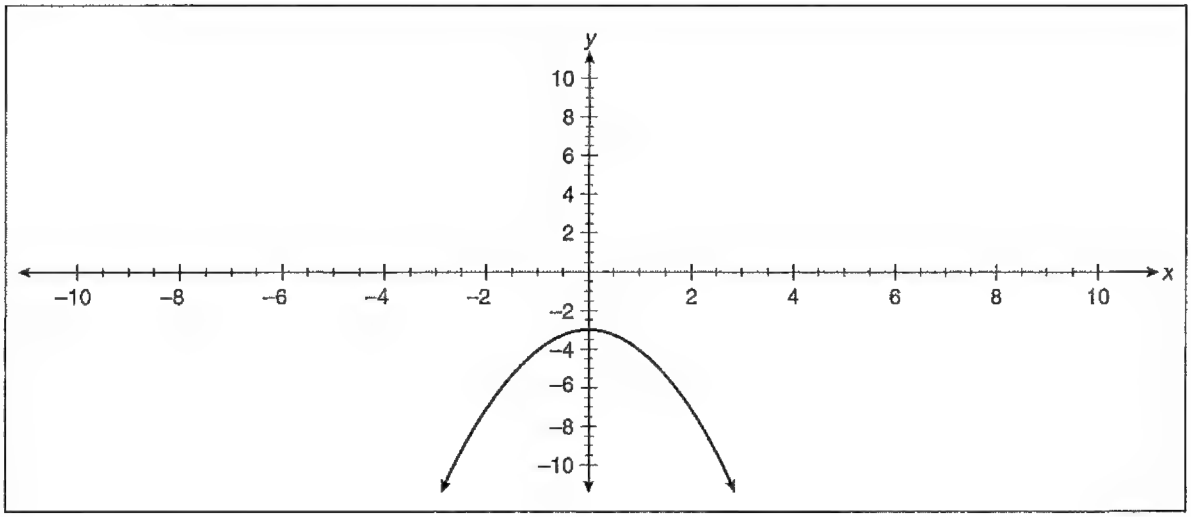


FIGURE 16.2



common denominators are easier to find, and reducing fractions is a snap. You will see how to add and subtract algebraic fractions first, before learning how to multiply and divide them.

Addition and Subtraction of Algebraic Fractions. These are the guidelines for working with fractions as you know them. The rule in the most basic language is this: When the bottoms are the same, you can add or subtract the tops. If the bottoms are not the same, you must find a new bottom. Mathematically it can be stated: When the denominators are the same, you may add or subtract the numerators. If the denominators are not the same, *then you must find a common denominator.* If you know the concept of fractions of one and can count, you have all the skills needed to add or subtract fractions.

Let's see how these guidelines work within algebra. Remember that addition and subtraction of fractions set up in the same way, so only one sample will be developed. Consider this problem:

$$\frac{x+5}{3x} + \frac{x+3}{x+1}$$

The common denominator would be ~~3(x + 1)~~ 3x(x+1). Multiply each numerator by what is NOT included in the common denominator.

$$\frac{(x+1)(x+5)}{3x(x+1)} + \frac{(3x)(x+3)}{3x(x+1)}$$

You will see that you can rewrite the problem to make your life easier. Multiply as needed.

$$\frac{(x^2+6x+6)}{3x^2+3x} + \frac{(3x^2+9x)}{3x^2+3x}$$

Combine like terms as you add.

$$\frac{4x^2+15x+6}{3x^2+3x}$$

This is the end result.

The same process would occur with subtraction, except you would be subtracting the term. Be very careful to watch out for sign changes, since we are defining a "minus sign" as "plus the opposite of whatever comes next."

Use your knowledge gained from the addition problem above to do this problem. As you write each step, explain why you are doing it.

$$\frac{x+5}{3x} - \frac{x+3}{x+1}$$

 $\frac{-2x^2-3x+5}{3x^2+3x}$

Your answer should be ~~$-2x^2 - 3x + 5$.~~ Here are the steps to solution. Compare the steps shown with the explanation above.

$$\frac{x+5}{3x} - \frac{x+3}{x+1}$$

$$\frac{(x+5)(x+1) - (3x)(x+3)}{3x(x+1)}$$

$$\frac{(x^2+6x+5) - (3x^2+9x)}{3x(x+1)}$$

$$\frac{x^2+6x+5-3x^2-9x}{3x^2+3x}$$

$$\frac{-2x^2-3x+5}{3x^2+3x}$$

Multiplication of Algebraic Fractions. Remember how we multiply and cancel with regular fractions:

- Multiplying fractions is simple: *top × top and bottom × bottom.*
- You will recall that when we reduced fractions, we factored the numerator and denominator and "canceled" any pairs of numbers found on the top and the bottom. When we multiply fractions, we can apply the same principle to make them easier to work with.
- Remember, when we reduce we factor out and remove fractions of ONE.

$$\frac{3x+15}{x+3} \times \frac{9x+27}{x+5}$$

$$\frac{3x+15}{x+3} \times \frac{9x+27}{x+5}$$

First, factor everything you can to see if you can cross-cancel. Note that 3 is the common factor in the first term and 9 is the common factor in the second. See in the next step how the factoring is shown.

$$\frac{3(x+5)}{x+3} \times \frac{9(x+3)}{x+5}$$

Look for the fractions of ONE: $\frac{x+3}{x+3}, \frac{x+5}{x+5}$.

$$\frac{3}{1} \times \frac{9}{1} = \frac{27}{1} \text{ or } 27$$

Write what remains and multiply.

$$\frac{8x+24}{2x+10} \times \frac{x+7}{3x+9}$$

After you cross-canceled and multiplied your result should have been

$$\frac{4(x+7)}{3(x+5)} \text{ or } \frac{4x+28}{3x+15}$$

The solution should look like this:

$$\frac{8x+24}{2x+10} \times \frac{x+7}{3x+9}$$

$$\frac{8(x+3)}{2(x+5)} \times \frac{x+7}{3(x+3)}$$

The $(x + 3)$ and the 8 and 2 cross cancel.

$$\frac{4}{x+5} \times \frac{x+7}{3}$$

$$\frac{4(x+7)}{3(x+5)} \text{ or } \frac{4x+28}{3x+15}$$

FIGURE 16.3

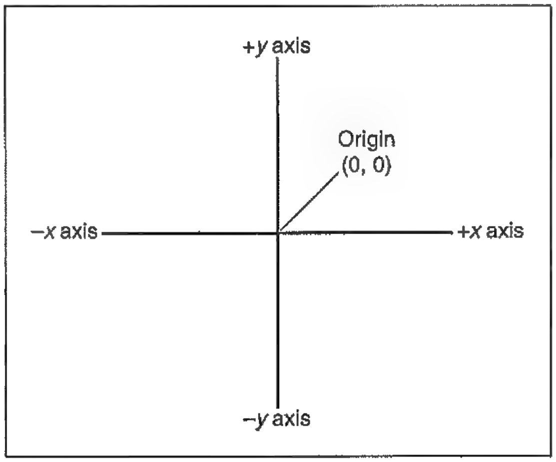
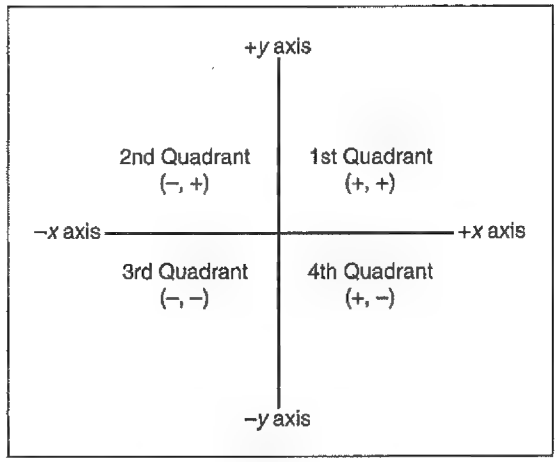


FIGURE 16.4



first coordinate is referred to as the *x*, and you move left (negative) or right (positive) from the origin, where the *x* and *y* axes intersect. The second coordinate is called the *y*, and it tells you where to move from the *x* coordinate. If it is negative you move down and if positive you move up.

Finding Line Equations

To find the equation of a line is a more complicated process. To start demystifying it, you need to understand the concept of slope. *Rise* is how far the one moves up or down from a point on the graph and *run* is the distance one moves left or right from that point. A slope can be positive, negative, or zero. It's easier to visualize it. Imagine you are driving a car. There are only so many directions one can drive in the real world. You can drive uphill, downhill, or on level ground. You will need to imagine driving from left to right. If you are driving uphill the slope is positive. If you are driving downhill left to right, the slope is negative. If you are driving on flat ground the slope is 0. If you find yourself driving vertically you are in real trouble, and the slope is declared as undefined.

Each point on a graph has a pair of coordinates. These coordinates are like directions given to drive somewhere in your car. The first direction (coordinate) given is always the *x*, and the second coordinate is always the *y*. When you have two in the same graph and a line runs through them, you can determine the slope of the line using the coordinates of those points. We will call them: (x_1, y_1) and (x_2, y_2). The subscripts are nothing mysterious. They designate only a position. So x_1 means that we are talking about the *x*-coordinate in the first pair, and y_2 means that we are discussing the *y*-coordinate in the second pair.

With this information we can expand the slope formula. Since the rise is the change (difference) in the *y*-coordinates, and the run is the change (difference) in the *x*-coordinates.

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Find the slope of a line passing through the points (1, −3) and (4, 3).

$$\text{Slope} = \frac{3-(-3)}{4-1}$$

$$\text{Slope} = \frac{3+3}{4-1}$$ (Remember how to subtract negatives?)

$$\text{Slope} = \frac{6}{3} = \frac{2}{1} \text{ or } 2$$

So the rise is 2 and the run is 1. In other words, from any point on the line, every time you go up 2 right, 1 you will hit the line again. When given a whole number as a slope place it over one to test it. For example, if they give you a slope of –6, put it over one: $-\frac{6}{1}$. Now compare it to the picture they give you. From a point on the line, if you go down 6 and right 1 do you hit the line again? If so, that is the correct slope. Now test your skill.

What is the slope of the line that passes through (–1, 4) and (3, 0)?

Your solution should be –1. If not, check your placement of number according to the slope formula:

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-4}{3-(-1)} = \frac{-4}{4} = \frac{-1}{1} \quad or \quad -1$$

With the slope of the line and the point where the line intersects (crosses) the *y-axis,* some very helpful equations develop that can make our life much easier. They are Slope–Intercept, Point–Slope, and Standard form. We choose the form that is easiest given the information and we can move from one form to another.

Slope–Intercept Form of the Equation. Use this pattern if you are given the slope of the line and the y-intercept.

The master pattern for Slope–Intercept is $y = mx + b$

m is the slope of the line.
b is the point where the line crosses the *y-axis.*

Working with these equations is simply a matter of substitution and solving. You have all the skills to do this.

Given $m = 6$ and $b = -4$, write the equation of the line in Slope–Intercept form.

$y = mx + b$	Write the master pattern for Slope–Intercept.
$y = 6x + (-4)$	Substitute the values of m and b into the pattern.
$y = 6x - 4$	Resolve any problems with addition of negatives.

Given $m = 5$ and $b = -8$, write the equation of the line in Slope–Intercept form.

After you substituted, you should have the equation $y = 5x - 8$. If not, go back and double-check your substitution.

$y = mx + b$
$m = 5$, place it in the m-position, and $b = -8$, place it in the b-position.
$y = 5x - 8$

Point–Slope Form of the Equation. Use this pattern if you are given the slope of a line and one point. This again is a matter of substitution.

The master pattern for Point–Slope is $y - y_1 = m(x - x_1)$

m	stands for the slope of the line.
(x_1, y_1)	is coordinate of a point on the line.
x and y	are the same variables as used in Slope–Intercept form.

(4,2)

If you are given a slope of –3 (m) and a coordinate of ~~2, 4~~ ~~(x_1, y_1)~~, you can write the equation of the line using the Point–Slope pattern, $y - y_1 = m(x - x_1)$. Simply substitute.

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -3(x - [2])$$
4
$$y - 2 = -3(x - 4)$$

Given the point (–4, 2) and the slope 2, determine the equation of the line in Point–Slope form.

$y - y_1 = m(x - x_1)$	Copy the master pattern for Point–Slope.
$y - 2 = 2(x - [-4])$	Substitute the values of the point and the slope in the appropriate positions.
$y - 2 = 2(x + 4)$	Before distributing, resolve any problems with addition or subtraction of negatives.

Given the point (–5, –7) and the slope –2, determine the equation of the line in Point–Slope form.

The equation in Point–Slope form would be $y + 7 = -2(x + 5)$. If you answer differs, let's solve it together.

$$y - y_1 = m(x - x_1)$$
$$m = -2$$
$$(x_1, y_1) = (-5, -7)$$
place the numbers in the Point-Slope formula
$$y + 7 = -2(x + 5)$$

The Distance Formula. You have seen that to use the Slope, Point–Slope and Slope–Intercept formulas you plugged the coordinates in where they belonged and things happened in the appropriate order. There are other uses for the coordinates (x_1, y_1) and (x_2, y_2). You can discover the distance of a line and the midpoint of a line using the appropriate formula. These formulas are easy to use. They are what are referred to in this book as "plug and play" formulas.

The distance formula will give you the "length" or distance a line travels on a graph.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

To use this formula, first put the coordinates in the appropriate places; second, use the order of operations to:

- Subtract the coordinates.
- Square the differences you have found.
- Add the results of the squaring.
- Take the square root of the result.

Let's use the coordinates (–2, 3) and (4, 2). Remember these numbers correspond to (x_1, y_1) and (x_2, y_2) respectively, so:

x_1 *is* –2
y_1 *is* 3
x_2 *is* 4
y_2 *is* 2

First, plug the numbers into the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(4 + 2)^2 + (2 - 3)^2}$$
$$d = \sqrt{(6)^2 + (-1)^2}$$
$$d = \sqrt{36 + 1}$$
$$d = \sqrt{37}$$

The distance is the $\sqrt{37}$.

Now to practice distance formula. Remember this is a plug and play situation. Put the numbers where they belong in the pattern and then crunch numbers.

Find the length of a line that passes from (3, 2) and (4, 6).

The solution is $d = \sqrt{17}$. Here is how the solution looks:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(4 - 3)^2 + (6 - 2)^2}$$
$$d = \sqrt{(1)^2 + (4)^2}$$
$$d = \sqrt{1 + 16}$$
$$d = \sqrt{17}$$

The Midpoint Formula. The midpoint formula will give you the coordinates of the middle of a line on a graph.

~~288 inches~~ $\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

To use this formula, first put the coordinates in the appropriate places; second, use the order of operations:

- Add the coordinates.
- Divide the sum by 2.

Now let's plug and play with the midpoint formula using the same coordinates: (–2, 3) and (4, 2).

$$\text{midpoint } \cancel{m} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$
$$\text{midpoint } \cancel{m} = \left(\frac{-2 + 4}{2}, \frac{3 + 2}{2}\right)$$
$$\text{midpoint } \cancel{m} = \left(1, \frac{5}{2}\right)$$

Find the midpoint of a line that passes from (3, 2) and (4, 6).

The midpoint of the line is the point with the coordinates $m = \left(\frac{7}{2}, 4\right)$

Let's examine how that solution was reached:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = \left(\frac{3+4}{2}, \frac{2+6}{2}\right)$$

$$M = \left(\frac{7}{2}, \frac{8}{2}\right)$$

midpoint $M = \left(\frac{7}{2}, 4\right)$

Graphing a One-Step Inequality. Graphing a one-step inequality works like graphing a regular linear equation with a small difference. That difference is that you will identify a whole group of answers by shading the appropriate area that describes the function of the equation. Consider the equation $y \geq x - 3$. What it states is that y will either be equal to $x - 3$ or greater than $x - 3$. To graph this equation, you would use a very quick trick that will identify where the line crosses the x-axis and the y-axis.

Find the x and y intercept for this equation: $y \geq x - 3$.

First, find the x-intercept (where it crosses the x-axis) by following these steps:

1. Make $y = 0$.
2. Do the calculation(s) and write in terms of x.
3. Put that point on the x-axis.

Plug 0 into the equation, $y \geq x - 3$, to find where the line crosses the x-axis. Since $y = 0$, the equation will now look like this:

$$0 \geq x - 3$$

Then solve for x as you did previously, and read the inequality from the variable.

$$3 \geq x$$

Note on the graph in Figure 16.5 how the line crosses the x-axis at 3.

Second, find the y-intercept (where it crosses the y-axis) by following these steps:

1. Make $x = 0$.
2. Do the calculation(s), and write in terms of y.
3. Put that point on the y-axis.

Plug 0 into the equation, $y \geq 0 - 3$, to find where the line crosses the y-axis. Since $x = 0$, the equation will now look like this:

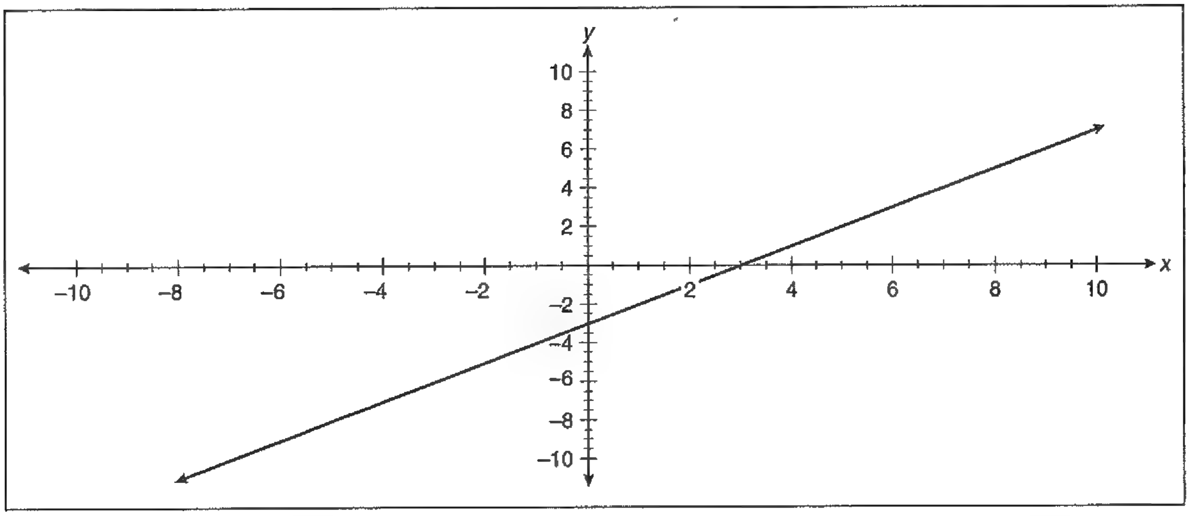
$$y \geq 0 - 3$$

Then solve for x as you did previously in Chapter 2, and read the inequality from the variable.

$$y \geq -3$$

Note on the graph in Figure 16.5 how the line crosses the y-axis at -3.

FIGURE 16.5



Third, graph the line defined by the x and y intercepts you placed on the graph. Follow these easy steps:

1. Draw a line extending through the two points you identified. You will use a solid line because you are using a ≤ or ≥. A solid line indicates that the line is part of the solution. If your equation had only a < or >, you would use a dotted line because it is not part of the solution.
2. Shade as necessary to adjust the graph.
   - If the symbol > is used, shade above the line.
   - If the symbol < is used, shade below the line.

In our equation $y \geq x - 3$ note how all possible answers would be ≥, so the graph would contain a solid line and be shaded above the line.

$y \leq x - 6$

This is how the inequality works out mathematically:

$y \leq x - 6$	
$0 \leq x - 6$	$y \leq 0 - 6$
$+6 \quad\quad +6$	$y \leq -6$
$6 \leq x$	$y \leq -6$

Your graph should look like Figure 16.6.

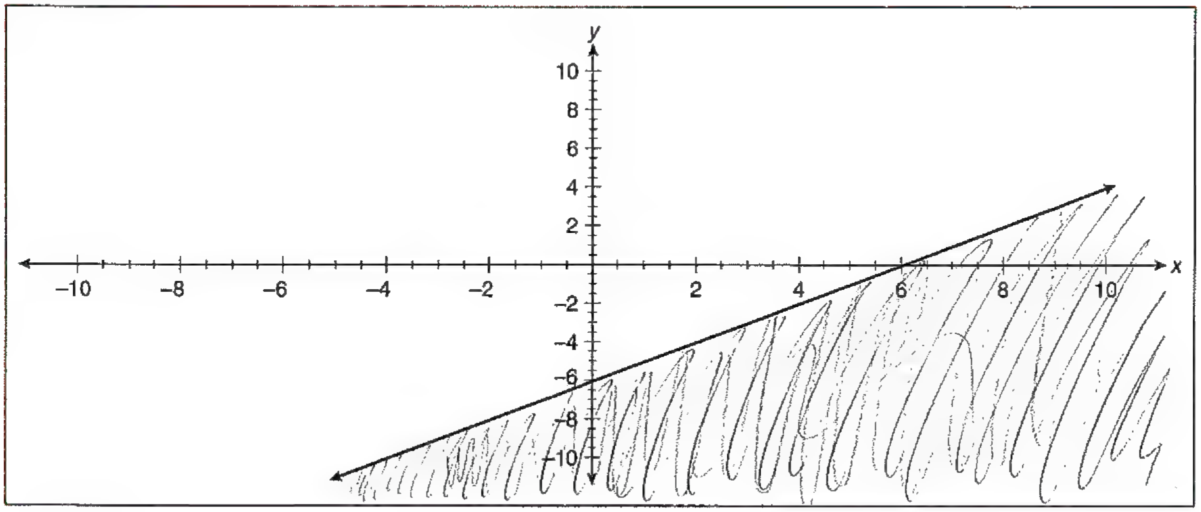
If you found your x-intercept as 6 and your y-intercept as −6, used a solid line and shaded below the line, you did it all correctly. If not, go back and review the steps presented above.

Finding the Equation of a Line If Two Points Are Known. If you are given only two points on a line such as (2, 3) and (6, 7) you can find the equation of the line. You will find this process very simple. It's just a matter of channeling information into the proper forms of the equation previously discussed. You will use the slope formula to find the slope of the line, and then either use the Slope–Intercept or Point–Slope patterns. Here are the steps to solution:

- Find the slope of the line using the slope formula.
- If they give you the y-intercept, use the Slope–Intercept pattern.
- If they do not give you the y-intercept, use the Point–Slope pattern.

10:20.

FIGURE 16.6



Standard Form of the Equation. If you have the Slope–Intercept or Point–Slope form of an equation, you can easily change it into standard form with the skills you have for moving things back and forth across the equals sign. You have the capability of doing this. You will simply move things back and forth across the equals sign using the inverse operations you used to solve equations.

The master pattern for standard form is $Ax + By = C$.

- A, B, C are real numbers with these exceptions:
  - A cannot be zero.
  - B cannot be zero.
- The x and y are the same ones from Slope–Intercept and Point–Slope forms.

Change from Slope–Intercept to standard form: $y = 6x$.

$y = 6x - 4$	To write in standard form, both variables must be on the left side
$-6x \quad -6x$	of the equals sign. Move the $6x$.
$y - 6x = -4$	Check to see that the x term comes first.
$-6x + y = -4$	

Change from Slope–Intercept to standard form: $y = -5x + 2$.

$5x + y = 2$

Did you get ~~$y = -5x + 2$~~? If so, you did all the steps correctly. If your answer differed, let's work it out together.

$$\begin{aligned} y &= -5x + 2 \\ +5x \quad & \quad +5x \\ y + 5x &= 2 \\ 5x + y &= 2 \end{aligned}$$

Change from Point–Slope to standard form: $y - 2 = 2(x + 4)$.

$y - 2 = 2x + 8$	First you must distribute.
$y - 2 = 2x + 8$	Collect like terms.
$+\,2 \qquad +2$	
$y = 2x + 10$	You now have the equation in slope–intercept form.

$\begin{aligned} y &= 2x + 10 \\ -2x \quad & \quad -2x \end{aligned}$	To write in standard form, both variables must be on the left side of the equals sign. Move the $2x$.
$\begin{aligned} y - 2x &= 10 \\ -2x + y &= 10 \end{aligned}$	Rearrange the variables in the appropriate order for standard form.
$\begin{aligned} (-1)(-2x + y &= 10) \\ 2x - y &= -10 \end{aligned}$	If you have a negative x term, multiply the whole equation through by -1 to resolve it.

Change from Point–Slope to standard form: $y + 6 = 3(x - 4)$.

Your standard form equation should be $3x - y = 18$. Here is the problem worked step by step for you:

$$\begin{aligned}
y + 6 &= 3(x - 4) \\
y + 6 &= 3x - 12 \\
-6 \quad & \quad -6 \\
y &= 3x - 18 \\
-3x \quad & \quad -3x \\
y - 3x &= -18 \\
-3x + y &= -18 \\
(-1)(-3x + y &= -18) \\
3x - y &= 18
\end{aligned}$$

CONTENT AREA FOURTEEN: INEQUALITIES

Inequalities

Inequalities, while they often operate like regular linear equations, use a specialized vocabulary and symbols.

$<$ Is less than.
$>$ Is greater than.
$\leq$ Is less than or equal to.
$\geq$ Is greater than or equal to.

Addition and Subtraction Properties. Just as there are addition and subtraction properties of equality, there are properties for the same operations involving inequalities.

The addition and subtraction properties can be stated as follows:

- Example One: Addition Property of Inequality

For all real numbers a, b, and c:	Example:
If $a > b$, then $a + c > b + c$	If $6 > 5$, then $6 + 3 > 5 + 3$
If $a < b$, then $a + c < b + c$	If $3 < 4$, then $3 + 1 < 4 + 1$

- Example Two: Subtraction Property of Inequality

For all real numbers a, b, and c	Example:
If $a > b$, then $a - c > b - c$	If $6 > 5$, then $6 - 3 > 5 - 3$
If $a < b$, then $a - c < b - c$	If $3 < 4$, then $3 - 5 < 4 - 5$

To solve an addition or subtraction inequality, you operate as you would for a linear equation using the inverse operation. Remember that addition and subtraction are inverse operations.

To undo addition we subtract, and to undo subtraction we add. Note below in the examples how the inverse operation is used as a method to isolate the variable and solve the inequality.

$$\begin{array}{rcl} a + 5 & < & 7 \\ -5 & & -5 \\ \hline a & < & 2 \end{array} \qquad \begin{array}{rcl} b - 7 & > & 9 \\ +7 & & +7 \\ \hline b & > & 16 \end{array} \qquad \begin{array}{rcl} 5 & \geq & c - 7 \\ +7 & & +7 \\ \hline 12 & \geq & c \end{array} \qquad \begin{array}{rcl} d + 9 & \leq & 3 \\ -9 & & -9 \\ \hline d & \leq & -6 \end{array}$$

Multiplication and Division of Inequalities. It is with the multiplication and division of inequalities that differences with linear equations begin to emerge. You will remember the rules of multiplication of signed numbers. When the signs are the same in multiplication and division the answer is positive, and when not the same the answer is negative. An examination of their properties will show the changes.

This example shows the properties of inequalities when the value of c is greater than zero. Note the examples carefully because of the contrast that will be pointed out in Example Four.

■ Example Three: Multiplication and Division Properties of Inequalities

(c is greater than 0)

For all real numbers a, b, and c ($c > 0$):	*Example:*
If $a > b$, then $ac > bc$	If $3 > 2$, then $3(4) > 2(4)$
If $a < b$, then $ac < bc$	If $1 < 4$, then $1(5) < 4(5)$
If $a > b$, then $\frac{a}{c} > \frac{b}{c}$	If $8 > 6$, then $\frac{8}{2} > \frac{6}{2}$
If $a < b$, then $\frac{a}{c} < \frac{b}{c}$	If $6 < 9$, then $\frac{6}{3} < \frac{9}{3}$

Example Four shows what happens in the multiplication and division of inequalities if the value of c is less than zero (a negative number). Note how the direction of the inequality reverses.

■ Example Four: Multiplication and Division Properties of Inequalities (c is less than 0)

For all real numbers a, b, and c ($c < 0$):	*Example:*
If $a > b$, then $ac < bc$	If $3 > -2$, then $3(-4) < -2(-4)$
If $a < b$, then $ac > bc$	If $-1 < 4$, then $-1(-5) > 4(-5)$
If $a > b$, then $\frac{a}{c} < \frac{b}{c}$	If $8 > -6$, then $\frac{8}{-2} < \frac{-6}{-2}$ or $-\frac{8}{2} < \frac{6}{2}$
If $a < b$, then $\frac{a}{c} > \frac{b}{c}$	If $-6 < 9$, then $\frac{-6}{-3} > \frac{9}{-3}$ or $\frac{6}{3} > -\frac{9}{3}$

You will notice that in some cases the sign of the inequality reverses.

To solve multiplication and division inequalities, you use the inverse operation. Remember that multiplication and division are inverse operations. To undo multiplication we divide, and to undo division we multiply. Note below in the examples how the inverse operation is used as a method to isolate the variable and to solve the inequality, being careful to remember that when multiplying or dividing by a negative you ~~sometimes~~ reverse the inequality. Consider the following:

Problem:	$3a < 9$	$\frac{b}{2} \geq 9$	$-5c > -25$	$\frac{d}{2} \geq 22$
Procedure:	divide 9 by 3	multiply by 2	divide by -5	multiply by 2
Answer:	$a < 3$	$b \geq 18$	$c < 5$	$d \geq 44$

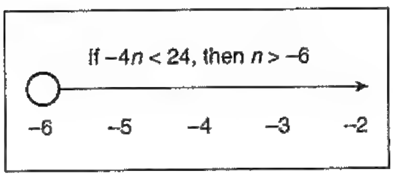
Solve: $-4n < 24$

You should have arrived at $n > -6$. If you have a different result, check against the solution below and watch and especially where the sign reverses when multiplying or dividing by a negative inequalities.

$$-4n < 24$$
$$\frac{-4n}{-4} < \frac{24}{-4}$$
$$n > -6$$

Graphing of One-Step Inequalities. Graphing of one-step inequalities uses the number line that you are already familiar with, but adds some new notation. A circle is used to indicate where one starts on the number line. An empty circle is used with < and >. The empty circle shows only the starting point and is not included in the solution. In the practice problem above, you would graph it on the number line placing an open circle on the –6 and the arrow would be drawn to the right (see Figure 16.7).

FIGURE 16.7



A solid circle is used with ≤ and ≥. The solid circle shows the that the number is included as part of the solution.

Solving Inequalities with Multiple Steps with One Variable. An inequality with multiple steps and one variable may sound immensely confusing, but look at the example below.

$$2(n + 6) > 18 - n$$

It looks just like a linear equation, and you have all the tools to solve it. One need only remember the steps to solving an equation because, with the exception of the sign reversal that occurs with multiplication and division of a negative number, they are the same. To prepare to solve this inequality, you will go through two stages: prep steps and solution steps. These are the same steps you would use to solve a regular equation.

- The Prep Steps:
  1. Distribute if necessary.
  2. Collect like terms on each side of the equals sign.
  3. Collect the variable on one side of the equals sign.
- The Solution Steps:
  1. Resolve any addition or subtraction using the inverse process.
  2. Resolve any multiplication or division using the inverse process, being careful with reversing of the sign if multiplying or dividing by a negative.

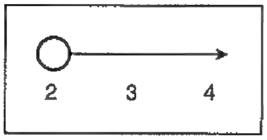
We will use the sample equation to illustrate the steps:

$2(n + 6) > 18 - n$ Prep Step: Distribute.

$\begin{array}{l} 2n + 12 > 18 - n \\ \underline{+n \qquad\qquad\quad + n} \end{array}$	Prep Step: Collect the variable on one side of the equals sign.
$\begin{array}{l} 3n + 12 > 18 \\ \underline{\quad -12 \;\; -12} \end{array}$	Solution Step: Undo the addition with subtraction.
$\dfrac{3n}{3} > \dfrac{6}{3}$	Solution Step: Undo the multiplication with division.
$n > 2$	

Graphing. Graphing follows the examples above using the number line (see Figure 16.8).

FIGURE 16.8



Solving Compound Inequalities with "And" and "Or." A compound inequality with "and" looks like this:

$-2 < n + 2 \leq 4$

A compound inequality with "or" looks like this:

$n - 4 \leq 3$ or $2n > 18$

A compound inequality is made up of two inequalities and is connected with either the word "and" or the word "or." To solve either one, you must separate the equations, then resolve them.

We shall tackle the "and" type using the sample problem above.

- **First,** separate $-2 < n + 2 \leq 4$ into two inequalities *reading from the center out.*

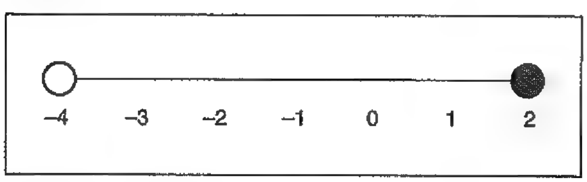
$n + 2 > -2$ and $n + 2 \leq 4$
$\underline{\quad -2 \quad -2}$ and $\underline{\quad -2 \quad -2}$

- **Second,** solve each separately using skills from Solving Inequalities to find that:

$n > -4 \qquad n \leq 2$

If you were asked to graph this compound inequality with "and," you would place a empty circle over −4 and a solid circle over 2 and connect the two symbols with a line (see Figure 16.9).

FIGURE 16.9



Compound Inequality with "And."

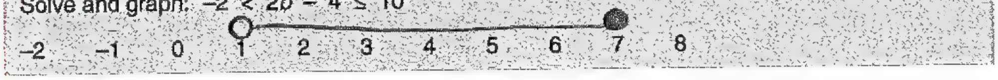
Solve and graph:	*Reasoning:*
$-1 < a + 3 < 7$	
$-1 < a + 3$ and $a + 3 < 7$	Separate the inequality into two parts reading from the center out.

$-4 < a$
and
$a < 4$

Solve each inequality separately. In this case subtracting 3 from each side.

To graph, you will use "open" circles because the signs do not include equality. Then draw a line between them.

Solve and graph: $-2 < 2b - 4 \leq 10$



In the practice problem $b > 1$ and $b \leq 7$. If you have a different answer, let's review the solution and the steps presented above.

$$\begin{array}{lcl}
-2 < 2b - 4 \leq 10 & & \\
-2 < 2b - 4 & \text{and} & 2b - 4 \leq 10 \\
-2 < 2b - 4 & \text{and} & 2b - 4 \leq 10 \\
+4 \qquad\quad +4 & & \quad +4 \quad +4 \\
2 < 2b & \text{and} & 2b \leq 14 \\
\frac{2}{2} < \frac{2b}{2} & \text{and} & \frac{2b}{2} \leq \frac{14}{2} \\
1 < b & \text{and} & b \leq 7
\end{array}$$

Did you split the inequality into two different parts $-2 < 2b - 4$ and $2b - 4 \leq 10$? Did you solve each separately? When you graphed, did you use an empty circle for $b > 1$ and a solid circle for $b \leq 7$? Did the line you drew cover only the space between the two?

Compound Inequality Using "Or." Next, we shall tackle the "or" type using the same sample problem.

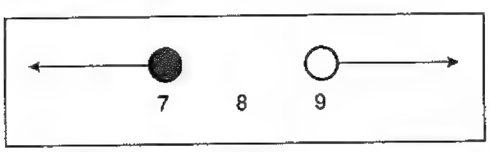
$n - 4 \leq 3$ or $2n > 18$

Solve each *separately* using skills from Solving Inequalities:

$$\begin{array}{lcl}
n - 4 \leq 3 & \text{or} & 2n > 18 \\
n - 4 \leq 3 & \text{or} & \frac{2n}{2} > 18 \\
\quad +4 \quad +4 & & \\
n \leq 7 & \text{or} & n > 9
\end{array}$$

If you were asked to graph this compound inequality with "or," you would place a solid circle over 7 and the arrow would go left, and place an empty circle over 9 and the arrow would go right. There would be no overlap of answers at all (see Figure 16.10).

FIGURE 16.10



Solve and graph: $3n + 1 < 4$ or $2n - 5 > 7$

$$\begin{array}{l}
3n + 1 < 4 \\
\underline{\quad\; -1 \quad -1} \\
\frac{3n}{3} < \frac{3}{3} \\
n < 1
\end{array}$$

Solve each for n, first adding or subtracting and then multiplying or dividing.

$$\begin{array}{r}2n - 5 > \ \ 7\\ \underline{\ \ +5 > +5}\end{array}$$

Solve each for n, first adding or subtracting and then multiplying or dividing.

$$\frac{2n}{2} > \frac{12}{2}$$

$$n > 6$$

$$n < 1 \text{ or } n > 6$$

To graph you will use open circles, since there is no equality indicated. Since this is an "or," the graph will move away from the points indicated.

Solve and graph: $n - 4 < -8$ or $n + 3 > 5$

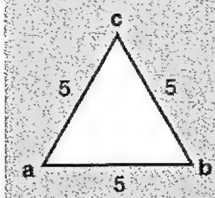
You should have arrived at $n < -4$ or $n > 2$. Here is the solution and some checkpoints to help you.

$$n - 4 \le -8 \quad \text{or} \quad n + 3 > 5$$

$$\begin{array}{rcl}n - 4 \le -8 & \text{or} & n + 3 > 5\\ +4 \quad +4 & & -3 \quad -3\\ n \le -4 & \text{or} & n > 2\end{array}$$

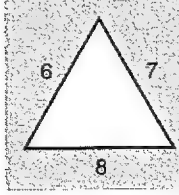
Did you solve each inequality separately? If you were to graph this answer on a number line, you would have a closed circle over −4 with an arrow going to the left and an open circle over 2 with an arrow to the right.

Perimeter of a Triangle:



Select angle "a"	Pick an angle. Starting at that angle, add the sides until you have completed a complete "walk" around the triangle.
5 + 5 + 5	
15	Write the sum of the sides.
15 units	Write with the appropriate unit used for measure. If no measurement is given, label it in units.

Practice Problem:



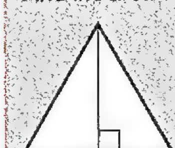
Your sum should be 21 units. Here is how the solution was reached:

Perimeter: Side 1 + Side 2 + Side 3
Perimeter = 6 + 7 + 8
Perimeter = 21

Area of a Triangle

Area: $A = \frac{1}{2}bh$ Multiply the base times the height and divide by 2. Label in square units.

Area of a Triangle:
Find the area of triangle abc:

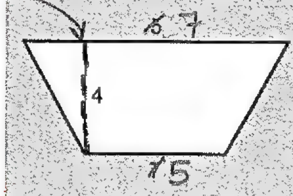


base = 4meters
height = 3meters

$A = \frac{1}{2}bh$	Write the formula.
$A = \frac{1}{2} \times 4 \times 3$	Substitute the values of b and h.
$A = 2 \times 3$	Take $\frac{1}{2}$ of the easiest number.
$A = 6$	Find the product.
$A = 6\text{m}^2$	Label the appropriate measure used making it squared. If no measure is given, write the answer in square units.

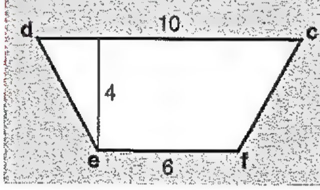
Find the area of trapezoid abcd:

dotted line



$A = \frac{1}{2}h(base_1 + base_2)$	Write the formula.
$A = \frac{1}{2}4(5 + 7)$	Substitute the values for h, $base_1$, and $base_2$.
$A = \frac{1}{2}(4)(12)$	Find the sum of the top and bottom base.
$A = \frac{1}{2}(4)(12)$	Take one half of the easiest number to cut in half.
$A = 2(12)$	Find the product.
$A = 24$ units^2	Label the appropriate measure used making it squared. If no measure is given, write the answer in square units.

Find the area of trapezoid dcef:



The area of trapezoid cdef is 32 square units. Let's examine the steps to solution of this area problem:

$$A = \frac{1}{2}h(base_1 + base_2)$$

$$A = \frac{1}{2}(4)(10 + 6)$$

$$A = (2)(16)$$

$$A = 32$$

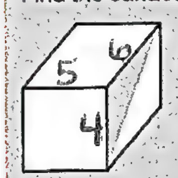
$$A = 32 \text{ units}^2$$

Surface Area of a Rectangular Prism. The great mathematician Polya once held up this shape to his graduate-level class and stated, "You look at this shape and think 'rectangular prism.' I call it a box." Take a clue from Polya and think "box." Before panic sets in, remember this is no more than a series of area problems with the answers added together. If you divide the box into its basic parts, the problem becomes easier. Each box has a front, back, top, bottom, left, and right side.

Top	base × height	(label in square units)
Bottom	base × height	(label in square units)
Front	base × height	(label in square units)
Back	base × height	(label in square units)
Left	base × height	(label in square units)
Right	+ base × height	(label in square units)
Total		(label in square units)

Imagine perfectly wrapping a present for a friend with the least amount of paper. That is the surface area.

Find the surface area of the rectangular prism:



It has a width of 5 inches, length of 6 inches, and height of 4 inches.

Top = base × height
Bottom = base × height
Front = base × height
Back = base × height
Left = base × height
Right = base × height

Write the formula for the top, bottom, front, back, left, and right sides.

Top = 5 × 6
Bottom = 5 × 6
Front = 5 × 4
Back = 5 × 4
Left = 4 × 6
Right = 4 × 6

Substitute in the appropriate values for the base and height of each side.

Top = 30
Bottom = 30
Front = 20
Back = 20
Left = 24
Right = 24

Find the product of each side.

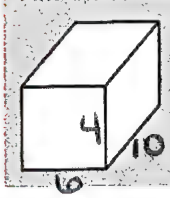
Surface Area = 148

Find the sum of all areas calculated.

Surface Area = 148 inches^2

Label the appropriate measure used making it squared. If no measure is given, write the answer in square units.

Find the surface area of this rectangular prism:



This figure has a base of 6 units, a height of 4 units, and a length of 10 units.

The surface area of the prism is 248 square units.
Let's walk through the solution step by step:

Top	6 × 10	=	60 square units
Bottom	6 × 10	=	60 square units
Front	6 × 4	=	24 square units
Back	6 × 4	=	24 square units
Left	10 × 4	=	40 square units
Right	+ 10 × 4	=	40 square units
Total		=	248 square units

$a^2 + b^2 = c^2$ Right where $a<c$ and $b<c$

$a^2 + b^2 < c^2$ Obtuse

$a^2 + b^2 > c^2$ Acute

Determine whether triangle abc is right, obtuse, or acute:

Leg a = 5

Leg b = 4

~~Hypotenuse~~ (c) = 6

$5^2 + 4^2 \ ? \ 6^2$	Leg squared + leg squared—Does it equal the ~~hypotenuse~~ c squared?
	Square the terms.
$\cancel{20}$ 25 + 16 ? 36	Add as needed.
$\cancel{36}$ 41 > 36	36 = 36, so the triangle is ~~a right~~ an acute triangle.

Determine whether triangle abc is right, obtuse, or acute:

Leg a = 7

Leg b = 3

~~Hypotenuse~~ (c) = 9

The triangle is obtuse. This is how that determination is made:

$a^2 + b^2 \ ? \ c^2$

$7^2 + 3^2 \ ? \ 9^2$

$49 + 9 \ ? \ 81$

$58 < 81 \qquad a^2 + b^2 < c^2$

Therefore, the triangle is obtuse.

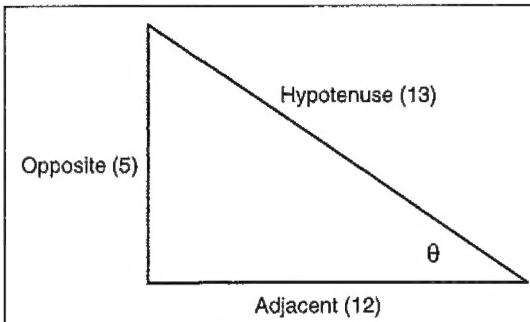
Trigonometry: A Quick Refresher

You may be asked questions on trigonometry, which is an offshoot of right-triangle relationships. The most common questions are plug and play ready. The three patterns that you will most likely see are *sine, cosine,* and *tangent.* These measurements relate an angle with two side of the triangle. They are not difficult to calculate, but you must understand how they are related to parts of a right triangle to be able to place the numbers properly. Let's look at those aspects first.

The symbol θ (greek *theta*) is often used to designate the angle. Don't let a symbol panic you; it's like using an x for an unknown quantity. The symbol designates which line is opposite and adjacent and which is the hypotenuse. If you move the symbol around on the triangle, you'll change what is opposite and what is adjacent.

Figure 17.3 shows the common formulae that you might encounter on the test.

FIGURE 17.3



Sine: $\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}}$

Cosine: $\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}}$

Tangent: $\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$

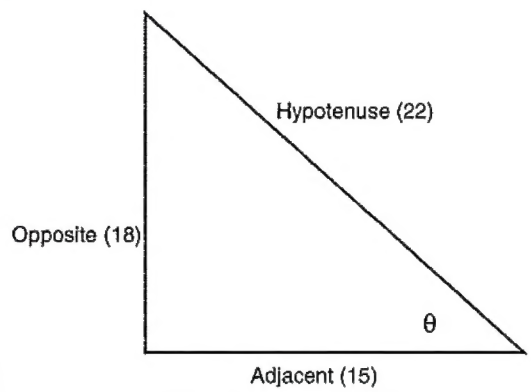
There is a memory device you can use to remember these relationships: SOH-CAH-TOA. Using the device, you can place the numbers in their proper places. Let's use the triangle in Figure 17.3 and see how it works (see Table 17.7).

TABLE 17.7

Sine	SOH	opposite/hypotenuse	$\frac{5}{13}$
Cosine	CAH	adjacent/hypotenuse	$\frac{12}{13}$
Tangent	TOA	opposite/adjacent	$\frac{5}{12}$

Let's use these relationships to find the sine, cosine, and tangent in the right triangle in Figure 17.4.

FIGURE 17.4



Solve each of items below:

Sine $\sin\theta = \frac{18}{22}$

Cosine $\cos\theta = \frac{15}{22}$

Tangent $\tan\theta = \frac{18}{15}$

You should have identified $\frac{18}{22}$ as the sine, $\frac{15}{22}$ as the cosine, and $\frac{18}{15}$ as the tangent. It's all in the placement!

Measurement in Standard and Metric Units

An understanding of U.S. standard and metric units is needed before any discussion of length, weight, or mass. Standard units are derived from early English units of measure. The yard was measured from the nose to finger tip, the inch from knuckle to finger tip, and the foot is obvious. All measurement is approximate. No ruler is accurate. As does time, standard measure has quirks that involve borrowing and carrying of units (see Table 18.2).

TABLE 18.2

1 mile	1760 yards
1 yard	3 feet
1 foot	12 inches
1 pound (lb)	16 ounces (oz)
1 ton (T)	2000 pounds (lbs)
1 gallon (gal)	4 quarts (qt)
1 quart (qt)	2 pints (pt)
1 pint (pt)	2 cups (cp)
1 cup (cp)	8 ~~16~~ ounces (oz)
1 gallon	768 teaspoons (tsp)
1 dozen	12 things
1 bushel	4 pecks

Metric Units of Measure. Metrics were devised as a method to bring commonality to units of measure and to remove the oddities that occur from country to country in national methods of measurement. They based this method on powers of 10. This would allow for increased commerce between nations. Metric measure has one standardized method no matter if you are working with length, weight, or liquid measure. It uses decimals in place of fractions for partial measures. Since metrics are still not standard measure in the United States, some approximations may be helpful:

- A meter is a bit more than a yard.
- A kilometer is about 3/5 of a mile.
- A kilogram is approximately 2.2 pounds.
- A liter is a little more than a quart.

Know the prefix and you are immediately oriented in the placement of numbers involved (see Table 18.3).

TABLE 18.3

kilo	thousand (1000)
hecto	hundred (100)
deka	ten (10)
meter	the basic unit of measure
deci	tenth (1/10)
centi	hundredth (1/100)
milli	thousandth (1/1000)

Note the standardization and how the use of fractions are avoided (see Table 18.4).

80 is related to "is"	Relate the numbers given to parts of the master pattern.
40 is associated with %	
What is matched with "of"	
$\frac{40}{100} = \frac{80}{?}$	Place the information we have using the master pattern.
$80 \times 100 = 40 \times ?$	Since it is a proportion we can cross-multiply the numbers.
$8000 \div 40 = 200$	Divide by the number across from the blank or variable.
Therefore 80 is 40% of 200.	

90 is 60% of what number?

The answer is 150. Did your answer differ? Let's examine the basics:

Did you set up a consistent relationship and place numbers in their proper positions?

$$\frac{part}{whole} = \frac{part}{whole} \qquad \frac{\%}{100} = \frac{is}{of} \qquad \frac{60}{100} = \frac{90}{?}$$

$90 \times 100 = 60 \times ?$
$9000 \div 60 = 150$
150

	What percent is 80 of 320?
$\frac{?}{100} = \frac{?}{?}$	Write the "bones" of the pattern.
80 is related to "is"	Relate the numbers given to parts of the master pattern.
320 is associated with "of"	
What is matched with %	
$\frac{?}{100} = \frac{80}{320}$	Place the information we have using the master pattern.
$100 \times 80 = 320 \times ?$	Since it is a proportion, we can cross multiply the numbers.
$8000 \div 320 = 25$	Divide by the number across from the blank or variable.
Therefore 80 is 25% of 320.	Be sure to label your answer as a %.

What percent is 75 of 250?

The correct answer is 30%. Did your answer differ? Let's examine the basics:

Did you set up a consistent relationship and place numbers in their proper positions?

$$\frac{part}{whole} = \frac{part}{whole} \qquad \frac{\%}{100} = \frac{is}{of} \qquad \frac{?}{100} = \frac{75}{250}$$

$100 \times 75 = 250 \times ?$
$7500 \div 250 = 30$
30%

Simple Word Problems with Percentages

Word problems aren't difficult just different. Since percentages are ratios and can also be in part/whole format, we can use them to solve many different types of word problems. Look for the key words "is" and "of," and if they cannot be found analyze the problem for something that can be put into part/whole format.

Consider this problem:

Seventy-five percent of the English class passed the test. If 15 students passed the test, how many students were in the class?